Word Problems: Mixture, Uniform, and Work Problems

I. Mixture Problems

Identify if it is a mixture problem: Mixture problems involves combining/mixing two ingredients/substances that have different characteristics and prices or percent into one mixture.

First step: Remember these formulas:

\[
\text{Amount} \times \text{Cost} \ (\$) = \text{Value} \ (A \times C = V) \quad \text{or}
\]
\[
\text{Amount} \times \text{Percent} \ (%) = \text{Value} \ (A \times P = V)
\]

Amount of the mixture is the sum of the amount of 2 ingredients/substances
Value of the mixture is the sum of the values of 2 ingredients/substances

HOWEVER, cost or percent of the mixture is NOT the sum of the cost of 2 ingredients

Second Step: Set up a table: let the unknown variable be \(x\) → fill out the table based on the given information and \(x\) → Set up an equation based on the table → Solve for \(x\)

Example: How many pounds of peanuts that cost $2.25 per pound must be mixed with 40lb of cashews that cost $6.00 per pound to make a mixture that costs $3.50 per pound?

We are given: Cost of peanuts = $2.25; Cost of cashews = $6.00; Cost of mixture= $3.50; Amount of cashews = 40 lbs.

Let the amount of peanuts (what we are looking for) be \(x\)

\[
\text{Amount of mixture} = \text{amount of peanuts} + \text{amount of cashews} = x + 40
\]

<table>
<thead>
<tr>
<th></th>
<th>Amount (A)</th>
<th>Cost (C)</th>
<th>Value (V)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Peanuts</td>
<td>(x)</td>
<td>2.25</td>
<td>2.25 (x)</td>
</tr>
<tr>
<td>Cashews</td>
<td>40</td>
<td>6.00</td>
<td>6.00 ((40))</td>
</tr>
<tr>
<td>Mixture</td>
<td>(x + 40)</td>
<td>3.50</td>
<td>3.50 ((x+4))</td>
</tr>
</tbody>
</table>

Because value of mixture = value of cashews + value of peanuts, we can get the following equation:

\[
2.25x + 6.00(40) = 3.50 \ (x+40)
\]

\[
2.25x + 240 = 3.50x + 140 \quad \text{(Subtract 3.50x on both sides)}
\]

\[
-1.25x = -100 \rightarrow x = \text{the amount of peanuts} = 80 \text{ pounds}
\]
II. Uniform Motion/Rate Problems

Identify if it is uniform motion/rate problem. An object that moves at a constant rate is said to be uniform motion. Key words: rate, time, and distance

**First Step:** Remember the formulas:

\[ \text{Rate} \times \text{Time} = \text{Distance} \ (r \times t = d) \]

\[ t = d/r \text{ and } r = d/t \]

**Second step:** Set up a table: let the unknown variable be \( x \) → fill out the table based on the given information and \( x \) → Set up an equation based on the table → Solve for \( x \)

**Example:** Two planes are 1620 miles apart and are traveling toward each other. One plane is travelling 120 mph faster than the other plane. The planes meet in 1.5 h. Find the speed of each plane.

We are given: total distance = 1620 miles = distance plane 1 travels + distance plane 2 travels

Time plane 1 travels = Time plane 2 travels = 1.5 h

We are looking for: Speed or rate for each plane given that one is 120 mph faster than the other

Let \( x \) be the rate of plane 1 (slower plane)

\[ \text{The rate of plane 2 (faster plane)} = x + 120 \]

<table>
<thead>
<tr>
<th></th>
<th>Rate (r)</th>
<th>Time (t)</th>
<th>Distance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plane 1</td>
<td>( x )</td>
<td>1.5</td>
<td>1.5( x )</td>
</tr>
<tr>
<td>Plane 2</td>
<td>( x + 120 )</td>
<td>1.5</td>
<td>1.5 ((x+120))</td>
</tr>
</tbody>
</table>

Total distance = 1620 miles = distance plane 1 travels + distance plane 2 travels, we have equation:

\[ 1.5x + 1.5 \ (x+120) = 1620 \]

\[ 1.5x + 1.5x + (1.5) \ 120 = 1620 \ (\text{Distribute } 1.5 \text{ into } x \text{ and } 120) \]

\[ 3x + 180 = 1620 \ (\text{Combine like terms}) \]

\[ 3x = 1440 \ (\text{Subtract } 180 \text{ on both sides}) \]

\[ x = 480 \rightarrow \text{Plane 1 (slower plane) travels at the speed } 480 \text{ mph} \]

The rate of plane 2 (faster plane) = \( x + 120 = 480 + 120 = 600 \text{ mph} \)
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III. Work problems

Identify if it is work problem. Key words: alone, together.

In a work problem, the rates at which certain persons or machines work alone are usually given, and it is necessary to compute the rate at which they work together (or vice versa)

First Step: Remember the formula:

\[
\frac{1}{r} + \frac{1}{s} = \frac{1}{t}
\]

In which: \(r, s\) are, for example, the number of hours it takes Rae and Sam, respectively to complete a job when working alone

\[
\frac{1}{r}, \frac{1}{s}
\]

Jobs done by Rae and Sam in 1 hour respectively

\(t\) is the total number of hours that it takes Rae and Sam to complete a job together.

Example: If Machine X can produce 1000 bolts in 4 hours and Machine Y can produce 1000 bolts in 5 hours, in how many hours can Machines X and Y, working together at these constant rates, produce 1000 bolts?

Given: Machine X can finish the job alone in 4 hours; Machine Y can finish the job alone in 5 hours

Let \(t\) = number of hours two machines work together to complete the job, we have:

\[
\frac{1}{4} + \frac{1}{5} = \frac{1}{t}
\]

\[
\frac{5}{20} + \frac{4}{20} = \frac{1}{t}
\]

\[
\frac{9}{20} = \frac{1}{t}
\]

\[9t = 20\]

\[t = \frac{20}{9}\]

References - The following works were referred to during the creation of this handout: *The Official Guide for GMAT Review Quantitative Review (Wiley, 2015)*