Basic Rules for Algebra

I. Quadratic Equation:

The standard formula: \( ax^2 + bx + c = 0 \) (a, b, c are known values, \( a \neq 0 \); x is unknown variable)

Example: \( x^2 + 5x + 6 \) is a quadratic equation. \( a=1, b=5, c=6 \)

\( 3x^2 + 5x \) is a quadratic equation; \( a=3, b=5, c=0 \)

\( 5x + 7 \) is NOT a quadratic equation because \( a=0 \) (there is no \( x^2 \))

To solve quadratic equations:

1. Quadratic formula: \( x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \)

Example: \( x^2 - 5x + 6 = 0 \). \( a=1, b=-5, c=6 \)

\( x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4*1*6}}{2*1} \) so \( x=-2 \) or \( x=-3 \)

2. Factoring by grouping or Diamond Method

Find 2 numbers that have the Sum of \( b \) and the Product of \( a \) times \( c \). After you get the two numbers, you will get the factors \((x + \text{constant 1})*(x + \text{constant 2})\)

Example: \( x^2 - 5x + 6 = 0 \)

So we need to find 2 constants that have the sum of -5 and product of 6

\[
\begin{array}{c|c|c|c|c}
\text{Sum} & b=-5 & -2 & -3 & \text{Product} \\
\hline
\text{Product} & a^2c = 1*6 = 6 & \hline
\end{array}
\]

\( x^2 - 5x + 6 = 0 \)

\( (x + (-2))*(x + (-3)) = 0 \)

\( (x - 2)(x - 3) = 0 \)

\( x - 2 = 0 \) or \( x - 3 = 0 \)

\( x = 2 \) or \( x = 3 \)
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II. Special Factors

Difference of Two Squares
\[ x^2 - y^2 = (x - y)(x + y) \]
Example: \[ x^2 - 9 = (x - 3)(x + 3) \]

Difference of Two Cubes
\[ x^3 - y^3 = (x - y)(x^2 + xy + y^2) \]
Example: \[ x^3 - 8 = (x - 2)(x^2 + 2x + 4) \]

Sum of Two Cubes
\[ x^3 + y^3 = (x + y)(x^2 - xy + y^2) \]
Example: \[ x^3 + 8 = (x + 2)(x^2 - 2x + 4) \]

III. Binomial Theorem

Definition: a quick way of expanding a binomial that is raised to any positive integer power

\[ (x + y)^2 = x^2 + 2xy + y^2 \]
Example: \[ (x + 2)^2 = x^2 + 2(x)(2) + 2^2 = x^2 + 4x + 4 \]

\[ (x - y)^2 = x^2 - 2xy + y^2 \]
Example: \[ (x - 2)^2 = x^2 - 2(x)(2) + 2^2 = x^2 - 4x + 4 \]

\[ (x + y)^3 = x^3 + y^3 + 3xy^2 + 3x^2y \]
\[ + 27x + 9x^2 \]
Example: \[ (x + 3)^3 = x^3 + 3^3 + 3(x)(3)^2 + 3(x)^23 = x^3 + 27x + 27x + 9x^2 \]

\[ (x - y)^3 = x^3 - y^3 + 3xy^2 - 3x^2y \]
\[ + 27x - 9x^2 \]
Example: \[ (x - 3)^3 = x^3 - 3^3 + 3(x)(3)^2 - 3(x)^23 = x^3 - 27x + 27x - 9x^2 \]

IV. Logarithmic Rules

Definition: Logarithms are the opposite of exponentials. With \( a > 0 \) and \( a \neq 1 \)

\[ \log_a x = y \] is equivalent to \[ x = a^y \]

Example: \[ \log_2 3 = x \rightarrow x = 2^3 = 8 \]
Example: \[ 10^3 = 1000 \rightarrow \log_{10} 1000 = 3 \]

Common Logarithm:
\[ \log x = \log_{10} x \]

Natural Logarithm:
\[ \ln x = \log_e x \]
Basic Rules for Algebra

Properties of Logs

\[ \log_a 1 = 0 \]  
Example: \( \log_7 1 = 0 \) (because \( 7^0 = 1 \))

\[ \log_a a = 1 \]  
Example: \( \log_5 5 = 1 \) (because \( 5^1 = 5 \))

\[ \log_a a^x = x \]  
Example: \( \log_5 5^8 = 8 \) (because \( 5^8 = 5^8 \))

\[ a^{\log_a x} = x \]  
Example: \( 5^{\log_5 12} = 12 \)

Laws of Logarithms

\[ \log_a (AB) = \log_a A + \log_a B \]  
Example: \( \log_2 (6x) = \log_2 6 + \log_2 x \)
  Example: \( \log_4 2 + \log_4 32 = \log_4 (2\cdot32) = \log_4 64 = 3 \)

\[ \log_a (A/B) = \log_a A - \log_a B \]  
Example: \( \log_5 (x/3) = \log_5 x - \log_5 3 \)
  Example: \( \log_2 80 - \log_2 5 = \log_2 (80/5) = \log_2 16 = 4 \)

\[ \log_a A^c = c \log_a A \]  
Example: \( \log_3 x^2 = 2 \log_3 x \)
  Example: \( 3 \log_4 x = \log_4 x^3 \)

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References - The following works were referred to during the creation of this handout: Valle Verde Tutorial Support Service Handout.