Basic Rules for Algebra

I. Quadratic Equation:

The standard formula: \( ax^2 + bx + c = 0 \) (a, b, c are known values, \( a \neq 0 \); \( x \) is unknown variable)

Example: \( x^2 + 5x + 6 \) is a quadratic equation. \( a=1, b=5, c=6 \)

\( 3x^2 + 5x \) is a quadratic equation; \( a=3, b=5, c=0 \)

\( 5x + 7 \) is NOT a quadratic equation because \( a=0 \) (there is no \( x^2 \))

To solve quadratic equations:

1. Quadratic formula: \( x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \)

Example: \( x^2 - 5x + 6 = 0 \). \( a=1, b=5, c=6 \)

\( x = \frac{-5 \pm \sqrt{5^2 - 4*1*6}}{2*1} \) so \( x = -2 \) or \( x = -3 \)

2. Factoring by grouping or Diamond Method

Find 2 numbers that have the Sum of \( b \) and the Product of \( a \times c \). After you get the two numbers, you will get the factors \((x - \text{first number})(x - \text{second number})\)

Example: \( x^2 - 5x + 6 = 0 \)

So we need to find 2 numbers that have the sum of -5 and product of 6

\[
\begin{array}{c|c}
\text{Sum} = b &= 5 \\
\text{Product} &= 1 \times 6 = 6 \\
-2 & -3 \\
\end{array}
\]

\( (x - (-2))(x - (-3)) = 0 \)

\( (x + 2)(x + 3) = 0 \)

\( x + 2 = 0 \) or \( x + 3 = 0 \)

\( x = -2 \) or \( x = -3 \)
Basic Rules for Algebra

II. Special Factors

Difference of Two Squares

\[ x^2 - y^2 = (x - y)(x + y) \]

Example: \[ x^2 - 9 = (x - 3)(x + 3) \]

Difference of Two Cubes

\[ x^3 - y^3 = (x - y)(x^2 + xy + y^2) \]

Example: \[ x^3 - 8 = (x - 2)(x^2 + 2x + 4) \]

Sum of Two Cubes

\[ x^3 + y^3 = (x + y)(x^2 - xy + y^2) \]

Example: \[ x^3 + 8 = (x + 2)(x^2 - 2x + 4) \]

III. Binomial Theorem

Definition: a quick way of expanding a binomial that is raised to any positive integer power

\[ (x + y)^2 = x^2 + 2xy + y^2 \]

Example: \[ (x + 2)^2 = x^2 + 2(x)(2) + 2^2 = x^2 + 4x + 4 \]

\[ (x - y)^2 = x^2 - 2xy + y^2 \]

Example: \[ (x - 2)^2 = x^2 - 2(x)(2) + 2^2 = x^2 - 4x + 4 \]

\[ (x + y)^3 = x^3 + y^3 + 3xy^2 + 3x^2y + 27x + 9x^2 \]

Example: \[ (x + 3)^3 = x^3 + 3x^2 + 3x + 27x + 9x^2 = x^3 + 27x + 27x + 9x^2 \]

\[ (x - y)^3 = x^3 - y^3 + 3xy^2 - 3x^2y + 27x - 9x^2 \]

Example: \[ (x - 3)^3 = x^3 - 3x^2 + 3x - 27x + 9x^2 = x^3 - 27x + 9x^2 \]

IV. Logarithmic Rules

Definition: Logarithms are the opposite of exponentials. With \( a > 0 \) and \( a \neq 1 \)

\[ \log_a x = y \] is equivalent to \( x = a^y \)

Example: \[ \log_3 x = 3 \rightarrow x = 2^3 = 8 \]

Example: \[ 10^3 = 1000 \rightarrow \log_{10} 1000 = 3 \]

Common Logarithm: \[ \log x = \log_{10} x \]

Natural Logarithm: \[ \ln x = \log_e x \]
Basic Rules for Algebra

Properties of Logs

\[ \log_a 1 = 0 \]  
\[ \log_a a = 1 \]  
\[ \log_a a^x = x \]  
\[ a^{\log_a x} = x \]

Example: \( \log_7 1 = 0 \) (because \( 7^0 = 1 \))
Example: \( \log_5 5 = 1 \) (because \( 5^1 = 5 \))
Example: \( \log_5 5^8 = 8 \) (because \( 5^8 = 5^8 \))
Example: \( 5^{\log_5 12} = 12 \)

Laws of Logarithms

\[ \log_a (AB) = \log_a A + \log_a B \]  
\[ \log_a (A/B) = \log_a A - \log_a B \]  
\[ \log_a A^c = c \log_a A \]

Example: \( \log_2 (6x) = \log_2 6 + \log_2 x \)
Example: \( \log_4 2 + \log_4 32 = \log_4 (2\cdot32) = \log_4 64 = 3 \)
Example: \( \log_5 (x/3) = \log_5 x - \log_5 3 \)
Example: \( \log_2 80 - \log_2 5 = \log_2 (80/5) = \log_2 16 = 4 \)
Example: \( 3 \log_4 x = \log_4 x^3 \)

References - The following works were referred to during the creation of this handout: Valle Verde Tutorial Support Service Handout.

(510) 885-3674  
www.csueastbay.edu/scaastrc  
scaastrc@csueastbay.edu