Statistics: T-Tests

What is a T-Test?

Every t-test needs a hypothesis test, statistical test, p-value, and t value in the t table to make a decision. However, this handout focuses on recognizing several types of t-test for different situations. If you need more to learn more about hypothesis, p-value, and t value in t table combine this handout with other sources.

A t-test is an analysis of two population means through the use of a statistical examination. A t-test tests the difference between the sample means when the variances \((\sigma^2)\) of two populations are unknown.

Different t-test are used for different hypothesis:

- 1-sample t-test
- 2-sample t-test
- Paired t-test

One-Sample T-test

Use the 1-sample t-test to estimate the mean of a population and compare it to a target or reference value when you do not know the standard deviation of the population. Use this test to determine whether the mean of a group differs from a specified value.

For example, when the mean waiting time for all patients in a medical clinic is greater than a target wait time of, say, 15 minutes, based on a random sample of patients.

\[
t = \frac{\bar{y} - \mu_0}{s/\sqrt{n}}
\]

The numerator is the difference between the mean of your sample \((\bar{y})\) and the hypothesized mean of the population \((\mu_0 = 15)\). The denominator is \(s\) as a sample standard deviation divided by square root of \(n\), which is sample size.

Two Sample T-Test

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A two sample t-test determines whether the means of two independent groups differ.

The observations from the first sample must not have any bearing on the observations from the second sample. For example, test scores of two separate groups of students are independent, but before-and-after measurements on the same group of students are not independent, although both of these examples have two samples.

The 2-sample t-test also works well when the assumption of normality is violated, but only if the underlying distribution is not highly skewed. With non-normal and highly skewed distributions, it might be more appropriate to use a nonparametric test.

The variances of the two samples may be assumed to be equal or unequal.

- Unequal Variances:
  \[ t = \frac{\bar{y}_1 - \bar{y}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} \]

- Equal Variances:
  \[ t = \frac{\bar{y}_1 - \bar{y}_2}{s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \]

  Where \[ s_p = \frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1 + n_2 - 2} \]

Paired T-Test

This test determines whether the mean of the differences between two paired samples differs from 0 (or a target value).

The paired observations are dependent and also, a paired t-test does not require both samples to have equal variance.

For example, suppose managers at a fitness facility want to determine whether their weight-loss program is effective. Because the "before" and "after" samples measure the same subjects, a paired t-test is the most appropriate analysis.

\( \bar{d} \) is sample mean of differences, \( s^2 \) is sample variance of differences, \( n \) is sample size, and \( t \) is t-test for paired samples.
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Calculate the mean difference: \( \bar{d} = \frac{d_1+d_2+\cdots+d_n}{n} \)

Calculate the standard deviation difference: \( s = \sqrt{\frac{(d_1-d\bar{d})+(d_2-d\bar{d})+\cdots+(d_n-d\bar{d})}{n-1}} \)

Now, calculate t-test for sample differences:

\[
t = \frac{\bar{d}-0}{\sqrt{s^2/n}}
\]

References: The following works were referred to during the creation of this handout: MiniTab’s “Why Should I Use a Paired T-Test?”, Statistics Solutions “Paired T-Test Sample”, and Wikipedia’s “One Sample T-Test” Article.