One Sample Z-Test and Confident Interval For Estimating A Population Mean

Step 1: Verify Assumptions

a. The sample is obtained using simple random sampling.

b. Sample size must be \( n > 30 \), population distribution must be normal with known variance. If variance is unknown, it can be approximated by sample variance \( S \), then use Z-test.

Step 2: State the Hypothesis

Step 3: Calculate the Test Statistic

\[
z = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}} \quad \text{or} \quad z = \frac{\bar{x} - \mu_0}{s / \sqrt{n}} \quad \text{(for large sample and \( \sigma \) is unknown)}
\]

Step 4: Decision Rule

a. p-value approach. Compute p-value, Reject \( H_0 \) when p-value < \( \alpha \).

\[
\begin{array}{|c|c|}
\hline
\text{Type of hypothesis} & \text{P-value} \\
\hline
\text{Two sided: if } H_a : \mu \neq \mu_0 & \text{p-value} = 2 \cdot P( Z \geq | z | ) \\
\hline
\text{Left sided: if } H_a : \mu < \mu_0 & \text{p-value} = P( Z \leq z ) \\
\hline
\text{Right sided: if } H_a : \mu > \mu_0 & \text{p-value} = P( Z \geq z ) \\
\hline
\end{array}
\]

b. Critical value approach: Determine critical value(s) using \( \alpha \).

\[
\begin{array}{|c|c|}
\hline
\text{Type of hypothesis} & \text{Reject } H_0 \\
\hline
\text{Two sided: if } H_a : \mu \neq \mu_0 & | z | > Z_{\alpha/2} \quad \text{equivalent to } z > Z_{\alpha/2} \text{ and } z < - Z_{\alpha/2} \\
\hline
\text{Left sided: if } H_a : \mu < \mu_0 & z < - Z_{\alpha} \\
\hline
\text{Right sided: if } H_a : \mu > \mu_0 & z > Z_{\alpha} \\
\hline
\end{array}
\]
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Step 5: State the Conclusion

<table>
<thead>
<tr>
<th>Original Claim is $H_0$</th>
<th>Original Claim is $H_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reject $H_0$</td>
<td>There is sufficient evidence (at the $\alpha$ level) to reject the claim that ... .</td>
</tr>
<tr>
<td></td>
<td>There is sufficient evidence (at the $\alpha$ level) to support the claim that ... .</td>
</tr>
<tr>
<td>Do Not Reject $H_0$</td>
<td>There is not sufficient evidence (at the $\alpha$ level) to reject the claim that ... .</td>
</tr>
<tr>
<td></td>
<td>There is not sufficient evidence (at the $\alpha$ level) to support the claim that ... .</td>
</tr>
</tbody>
</table>

Note: The level of significance is used to determine the critical value. The critical region includes the values of the shaded region. The shaded region is $\alpha$. Using Z-table to find critical value.

Two types of errors in decision making

<table>
<thead>
<tr>
<th>Research Reality</th>
<th>Null $H_0$ is True = Ha is False</th>
<th>Null $H_0$ is False = Ha is True</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reject $H_0$</td>
<td>Type I error $\alpha$</td>
<td>Correct Decision</td>
</tr>
<tr>
<td>Reject Ha</td>
<td>Power = 1-$\beta$</td>
<td>Type II error $\beta$</td>
</tr>
</tbody>
</table>

Confidence interval: The $(1 - \alpha)\%$ confidence interval estimate for population mean is

$$\bar{x} \pm z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} \quad \text{or} \quad \bar{x} \pm z_{\alpha/2} \cdot \frac{s}{\sqrt{n}} \quad (\text{if large sample and unknown variance})$$